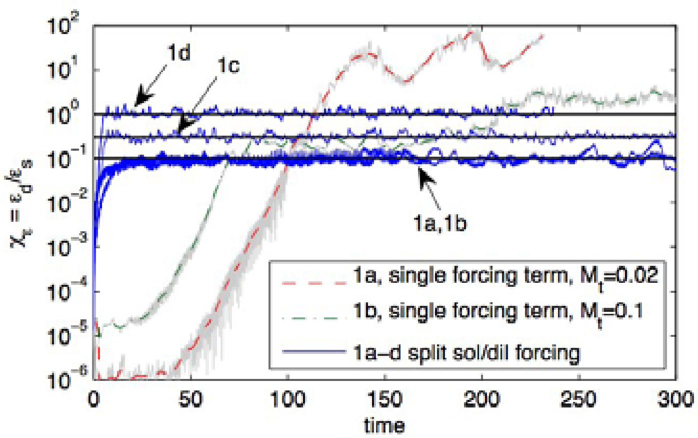
A Realistic and Stable Method of Forcing for Compressible Isotropic Turbulence

Mark R. Petersen, Daniel Livescu, Sumner Dean, Jamaludin Mohd-Yusof, CCS-2

Numerical simulations to investigate the behavior of isotropic turbulence play an important role in turbulence research, especially since the advances in supercomputing power now allow simulations at Reynolds numbers comparable with or even larger than those obtained in laboratory experiments. If a forcing term is added to the equations, then the flow could reach a statistically stationary state, where the injection rate (usually at large scales) is equal to the rate of energy dissipated at small scales. Forcing has several advantages—the Reynolds number of the simulation can be increased considerably, statistics can be averaged over time, decreasing statistical variability due to transient effects, and natural systems are usually forced.

Forcing in nature is due to large-scale effects, for example, solar-induced, buoyancy-driven convection in the atmosphere. The characteristics of turbulence at much smaller scales is thought to be independent of the nature of the forcing, which is why turbulence is often studied in an idealized, triply periodic domain.

Fig. 1. Evolution of the dilatational-to-solenoidal dissipation ratio, for single term and the new split forcing method. The single-term method is unstable, and dilatational energy can grow without bound. Four simulations with our proposed split forcing method show that statistics quickly adjust to the imposed value (horizontal black lines).



We have introduced a method of forcing for compressible, isotropic turbulence where one may precisely control the stationary-state values of the dissipation (and thus the Kolmogorov scale) and the ratio of dilatational (compressive) to solenoidal (rotational) energy [1]. This method is based on a linear-forcing method, where the forcing term is simply proportional to the momentum, that has been successfully implemented for incompressible flows [2,3].

Linear forcing is physically realistic because it is similar to the natural Reynolds shear stress production mechanism in the turbulent kinetic energy equation. The single-term linear forcing used in current incompressible simulations proved to be unstable for compressible flows because there is no control over the dilatational energy, which grows without bound (Fig. 1). In order to overcome this issue, we developed a two-term forcing method that controls the solenoidal and dilatational dissipation separately.

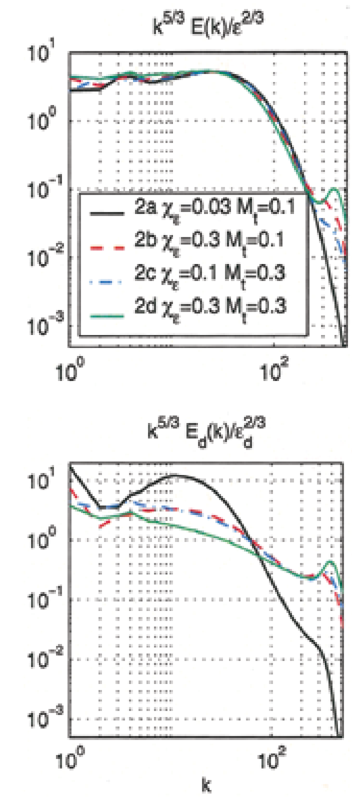


Fig. 2. Compensated energy spectra from four 1024^3 isotropic simulations: total kinetic energy (top), and its dilatational component (bottom).

Forcing may be applied only at low wavenumbers (large scales) or full spectrum (all scales). A comparison of these methods in the context of linearly forced compressible flows showed that low-wavenumber forcing achieves Reynolds numbers that are nearly twice as high, leading to increased range of dynamically relevant scales at the same resolution. The low-wavenumber method also allows us to avoid the computational cost of a full Fourier transform, as one may simply sum the low-wavenumber modes in spectral space.

We research the nature of compressible turbulence using direct numerical simulation (DNS), where all scales down to the viscous dissipation range are fully resolved [4]. The two-term linear, low-wave forcing method has allowed us to conduct a large study of $M_t - \chi$ parameter space, where Mt is the turbulent Mach number and χ is the ratio of dilatational to solenoidal kinetic energy. These parameters exhibit the effects of compressibility; low M_t, low χ simulations are nearly incompressible, while high M_t, high χ simulations have strong density gradients and shock waves. Long-time averages of statistics in M_t - χ parameter space show how these quantities vary. This type of data is useful for testing and developing models for the effects of compressibility in engineering-scale simulations where the smallest scales are not resolved.

For this study, four simulations with 1024^3 grid points were conducted on 1024 processors on Purple, a supercomputer at LLNL. These are the highest resolution compressible Navier-Stokes DNS to date, and achieve Taylor Reynolds numbers greater than 300. In addition, eighteen simulations with Taylor Reynolds numbers of ~100 were conducted on the San Diego Supercomputer Center's Blue Gene system. These simulations cover a range of M_t = 0.02-0.3 and χ = 0.0-1.0. Results show that the total energy spectrum is close to the Kolmogorov 1941 $k^{-5/3}$ law in the inertial range for all parameters, while the dilatational (compressive) spectrum is steeper than $k^{-5/3}$ (Fig. 2), but varies based on the parameters.

There are many differences between compressible turbulence and the more commonly studied incompressible turbulence. In many applications, such as ocean dynamics and large-scale atmospheric motion, the velocity fluctuations are small compared with the speed of sound, the density is nearly constant, and the irrotational effects are small. However, as the velocity fluctuations become comparable to the sound speed and/or exothermic reactions take place, new phenomena occur—shock waves, localized expansion and contractions providing a distinct dissipation mechanism, new nonlinearities leading to additional scale coupling, and strong fluctuations of the thermodynamic quantities. The characteristics of compressibility can be observed in visualizations of divergence and vorticity, where large variability and gradients can be seen (Figs. 3 and 4). Shocklets are short-lived, isolated shock waves that occur in these areas, and are thought to be responsible for many of the effects of compressible turbulence [5].

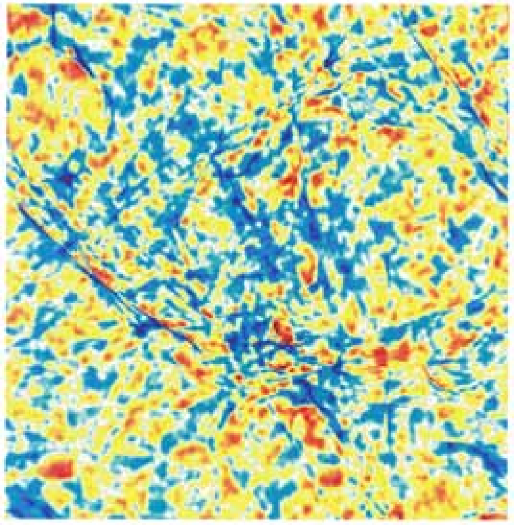


Fig. 3. Cross section of the divergence in a 1024^3 compressible isotropic simulation. Purple lines show shocks in the flow.

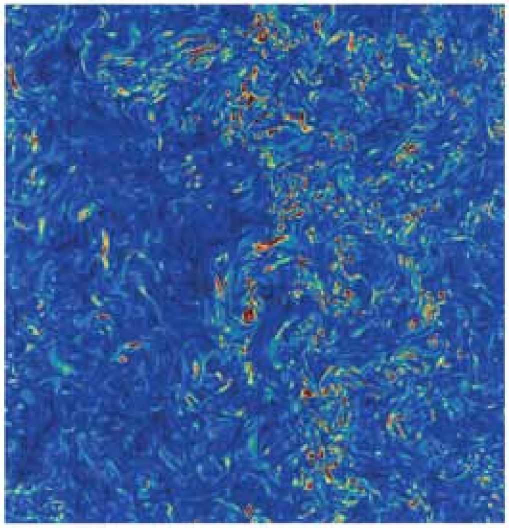


Fig. 4. Cross section of vorticity magnitude show an active eddy field.

For more information contact Mark Petersen at mpetersen@lanl.gov.

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